



Conic10K: A Challenging Math Problem Understanding and Reasoning Dataset

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What is Conic10K

Paper



Dataset 10861 challenging math problems



Conic Section

Focuses on conic sections in Chinese senior high school)non-onde

Open-ended

Consists of openended questions, no choices for random



Complex

Requires complex reasoning steps



Rationale

With high-quality formal representations and rationales



Serves as a benchmark for mathematical understanding and

Example		
Question: 点P(x,y)是椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$ 上的一点, F_1, F_2 是椭圆的两个焦点, 且 $\angle F_1 P F_2 \le 90^\circ$,则该椭圆的离心率的取值范围是? (Let P(x,y) be an arbitrary point on the $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$. F_1, F_2 are the two foci of the ellipse, $\angle F_1 P F_2 \le 90^\circ$. What is the range of values for the eccentricity of the ellipse?)	<pre>Formal Representation: P: Point PointOnCurve(P, G)=True Coordinate(P)=(x1, y1) x1,y1: Number</pre>	Span: 点P(x,y) 点P(x,y)是…一点 P(x,y) P(x,y)
<u>Rationale:</u>	G: Ellipse	椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$
由题意可知,当点P位于 (0,b) 或 (0,-b) 处时, $\angle F_1 P F_2 = 90^\circ$ 最大, 此时 $\cos \angle F_1 P F_2 = \frac{a^2 + a^2 - 4c^2}{2a^2} = \frac{a^2 - 2c^2}{a^2} \ge 0, a \ge \sqrt{2}$ 。因为 $e = c/a$,所以 $e \le \sqrt{2}$ 。因为 e 是椭圆离心率, $0 \le e \le 1$,所以 $0 \le e \le \sqrt{2}$ 。	<pre>Expression(G)=(y^2/b^2+x^2/a^2=1) a, b: Number</pre>	椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$ $x^2 + y^2(a > b > 0)$
(When the point <i>P</i> is located at $(0, b)$ or $(0, -b)$, the angle $\angle F_1 P F_2 = 90^\circ$ is at its maximum. In this case, $\cos \angle F_1 P F_2 = \frac{a^2 + a^2 - 4c^2}{2a^2} = \frac{a^2 - 2c^2}{a^2} \ge 0, a \ge \sqrt{2}$. Since $a = \frac{c}{a}$ we have $a < \frac{\sqrt{2}}{2}$. As <i>e</i> represents the eccentricity of the ellipse	a > D b > 0 F1, F2: Point	$\frac{1}{a^2} + \frac{1}{b^2}(a > b > 0)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2}(a > b > 0)$ F_1, F_2
and it lies within the range $0 < \rho < 1$ we can conclude that $0 < \rho < \frac{\sqrt{2}}{2}$	$Focus(G) = \{F1, F2\}$	F_1, F_2 是椭圆的两个焦点
$\frac{1}{2}$	AngleOf(F1,P,F2)<=Unit(90,degree)	$\angle F_1 P F_2 \leq 90^{\circ}$

<u>Answer:</u> $(0, \frac{\sqrt{2}}{2}]$

Reasoning

REASONING STEPS



The average number of reasoning steps* required by Conic10K is much more than that of other math datasets.

* We uniformly sampled 30 problems from each dataset and ask the two students to annotate the reasoning steps.

- Evaluation of existing models

Model	Avg. Acc.	# Syntax Err.
Full finetuning		
mT5-large	77.6	12
mT0-large	80.7	<u>6</u>
mT5-xl	82.6	9
mT0-xl	<u>84.6</u>	8
LoRA finetuning		
Bloomz-1b7	62.7	13
Bloomz-3b	67.6	6
Bloomz-7b1	71.3	4
Falcon-7b	58.0	10
LLaMA-7b	71.1	5
ChatGLM-6b	74.7	7
Vicuna-7b	76.9	3

Model	Avg. Acc.
Full finetuning	
mT0-xl	12.5
mT5-xl	14.8
LoRA Finetuning	
Vicuna-7b	17.9
ChatGLM-6b	22.5
Zero-shot CoT	
Vicuna-13b	0.9
ChatGLM-6b	1.5
GPT-3.5-turbo	6.2
GPT-3.5-turbo + ICL	5.9
GPT-3.5-turbo + SC	6.8
GPT-4	15.5
GPT-4 + En	26.0
References	
Guessing '2'	4.5
Human Expert	57.5

GPT-4 Case

USER Given a point *P* on the parabola with equation $y^2 = 4x$, let's denote the distance from point P to the yaxis as *d*. For a given point *A*(4,5), what is the minimum value of |PA| + d?



First, we can write the equation of the parabola as $y^2 = 4x$. Let's assume the coordinates of point *P* as (x_0, y_0) , so we have $y_0^2 = 4x_0$. The distance from point *P* to the y-axis is denoted as *d*, which is equal to $d = x_0$. We need to find the minimum value of |PA| + d, which is equivalent to finding the minimum value of $|PA| + x_0$. Using the distance formula between two points, we have $|PA| = \sqrt{(x_0 - 4)^2 + (y_0 - 5)^2}$. Substituting $y_0^2 = 4x_0$, we get $|PA| = \sqrt{(x_0 - 4)^2 + (4x_0 - 5)^2}$. Now we need to find the minimum value of ...(omit texts).... Therefore, the minimum value of |PA| + d is $\sqrt{\frac{144}{17}}$.

Table 14: Results on semantic parsing. # Syntax Err. means the number of syntactic errors in the generated expressions.

Table 13: Results on mathQA. ICL and SC mean in-context-learning and self-consistency. **GPT-4+En** means the problems are translated into English.

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* The prompts are omitted in this example. Starting from the red part, the response from GPT-4 is incorrect, so we omit most of its responses after that. We find that GPT-4 can hardly find the shortcut in the problem and make mistakes in complicated calculations.